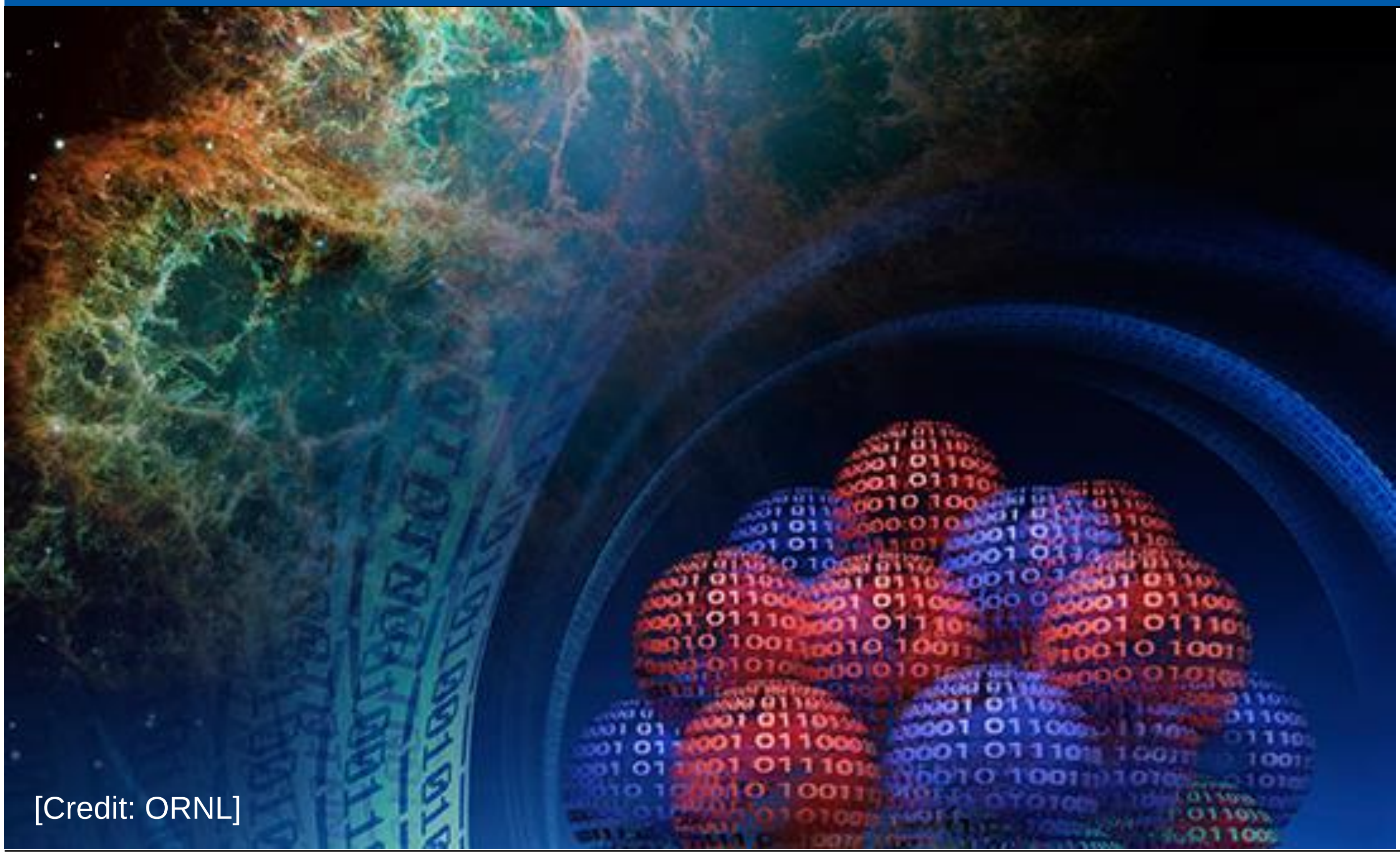
Applications of chiral nuclear forces up to N^{3}LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Christian Drischler

The 2017 ICNT Program at FRIB

April 5, 2017



[Credit: ORNL]

supported by:

SFB 1245

European Research Council
Established by the European Commission

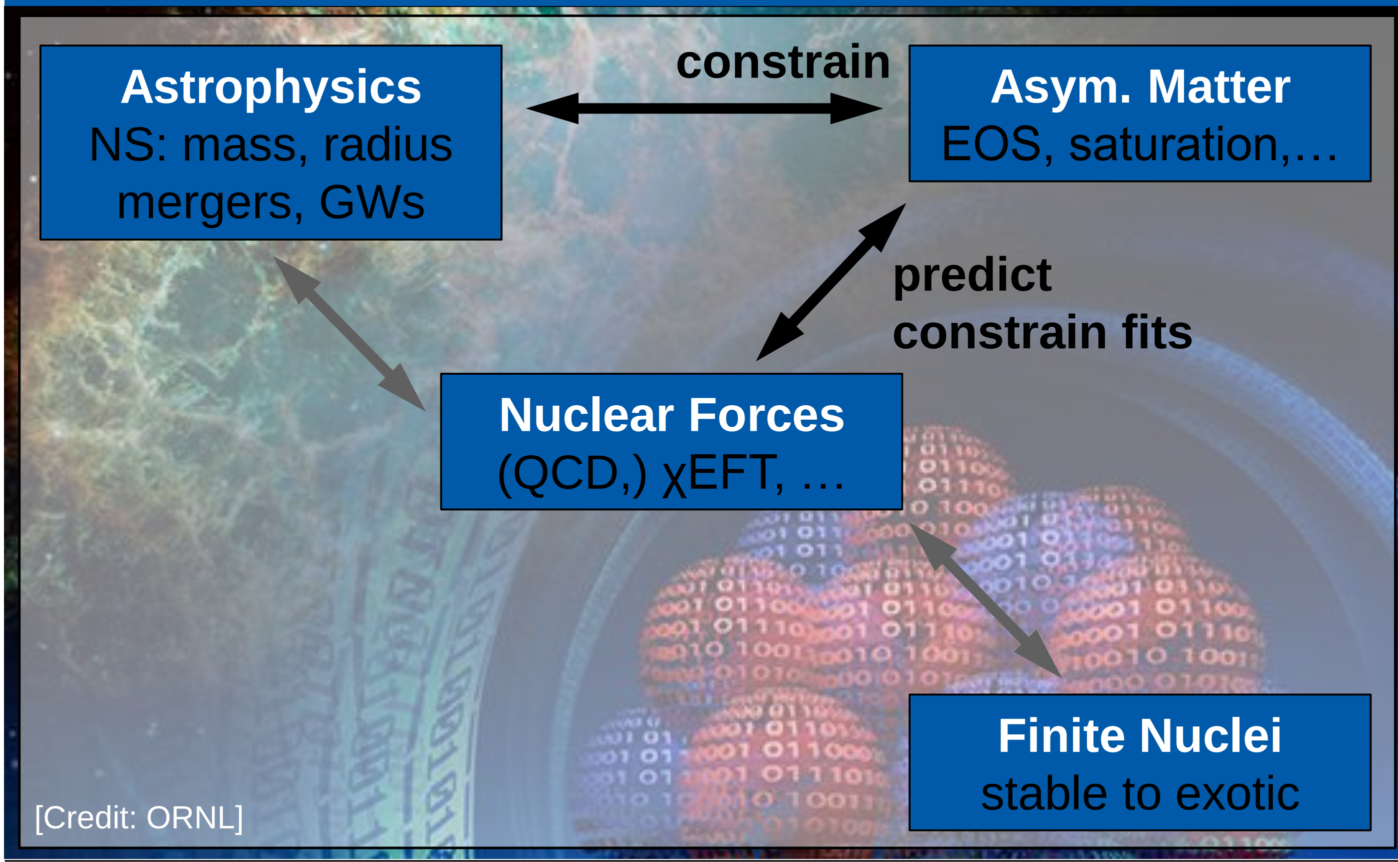
Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Christian Drischler

The 2017 ICNT Program at FRIB

April 5, 2017



supported by:

SFB 1245

European Research Council

Established by the European Commission

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Motivation: Infinite Matter

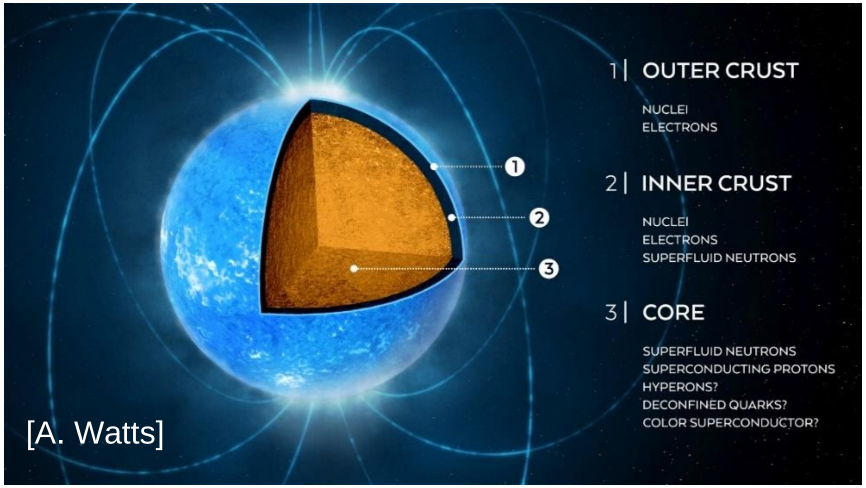
see also: Wellenhofer *et al.*, PRC **93**, 055802

Energy per particle: $\frac{E}{A}(n, x, T)$

density
proton fraction
temperature

based on **chiral effective field theory** (EFT):

- direct determination of astrophysical quantities: sym. energy, …
- ideal to **test** (and to improve) **nuclear forces** ~n_0
- constrain neutron-star EOS: mass-radius relations, …



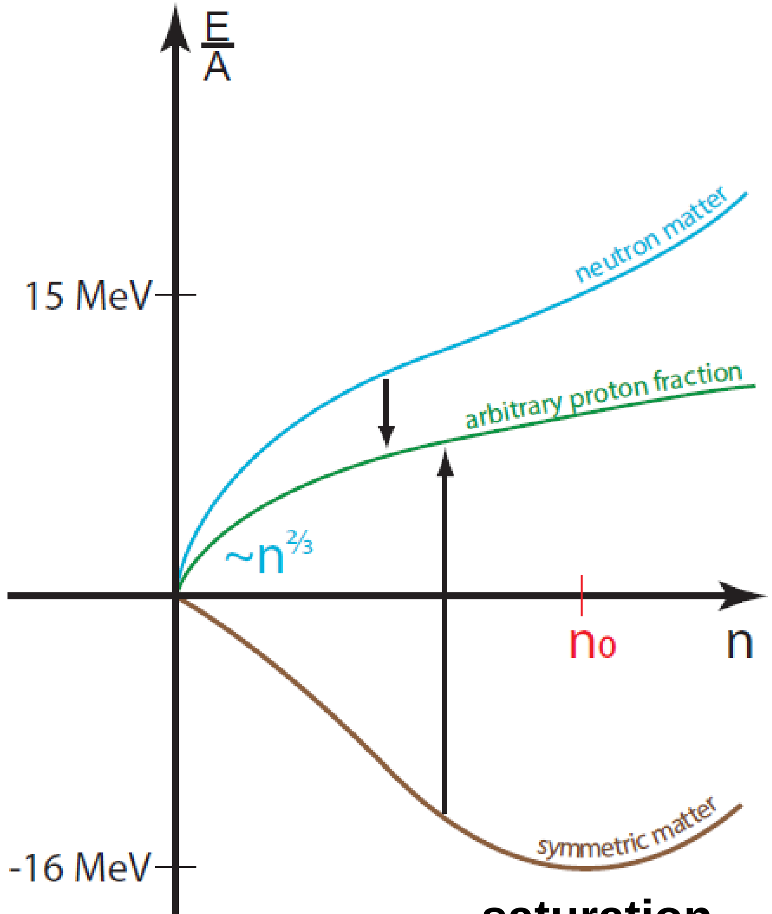
$$S_2(n) = \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\frac{E}{A}(n,\beta)\bigg|_{\beta=0}$$

$$L = 3n_0\frac{\partial}{\partial n}S_2(n)\bigg|_{n=n_0}$$

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Landscape of Nuclear Matter



saturation

neutron matter (*Z* = 0):

Hebeler *et al.*, Astrophys. J., **773**, 11 (2013)
Krüger *et al.*, PRC **88**, 025802 (2013)
Gezerlis *et al.*, PRL **111**, 032501 (2013)
Roggero *et al.*, PRL **112**, 221103 (2014)
Wlazłowski *et al.*, PRL **113**, 182503 (2014)
Lynn *et al.*, PRL **116**, 062501 (2016)
Dyhdalo *et al.*, PRC **94**, 034001 (2016)
. . .

symmetric matter (*N* = *Z*):

Hebeler *et al*. PRC **83**, 031301(R) (2011)
Holt, Kaiser, Weise, PPNP **73** 35 (2013)
Coraggio, Holt *et al*. PRC **89**, 044321 (2014)
Wellenhofer *et al.*, PRC **89**, 064009 (2014)
Carbone *et al*., PRC **90** 054322 (2014)
Coraggio, Holt *et al*., PRC **89**, 044321 (2014)
. . .

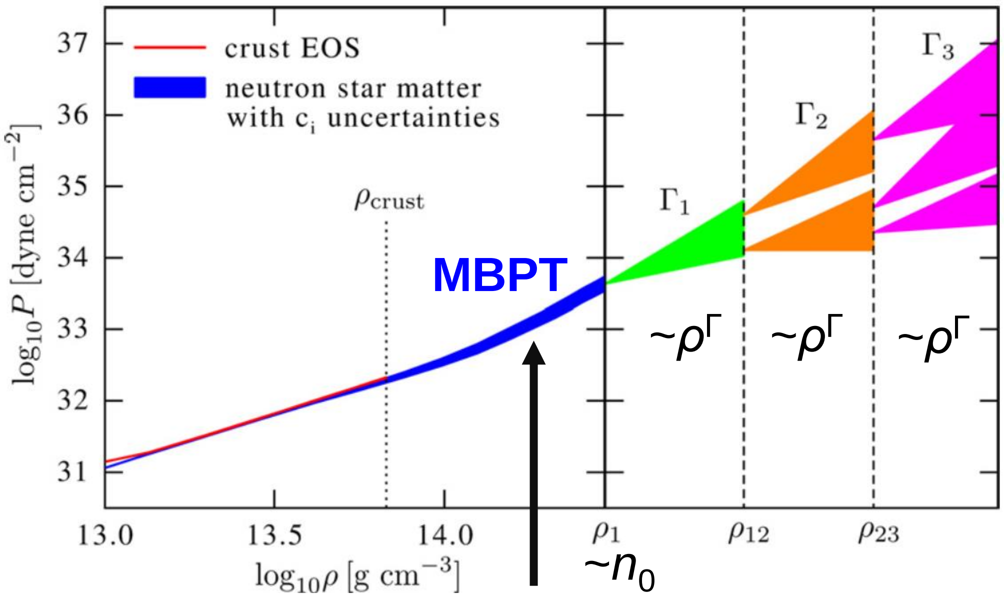
applications to neutron stars??

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Constraining Neutron Stars

Hebeler *et al.*, Astrophys. J., **773**, 11

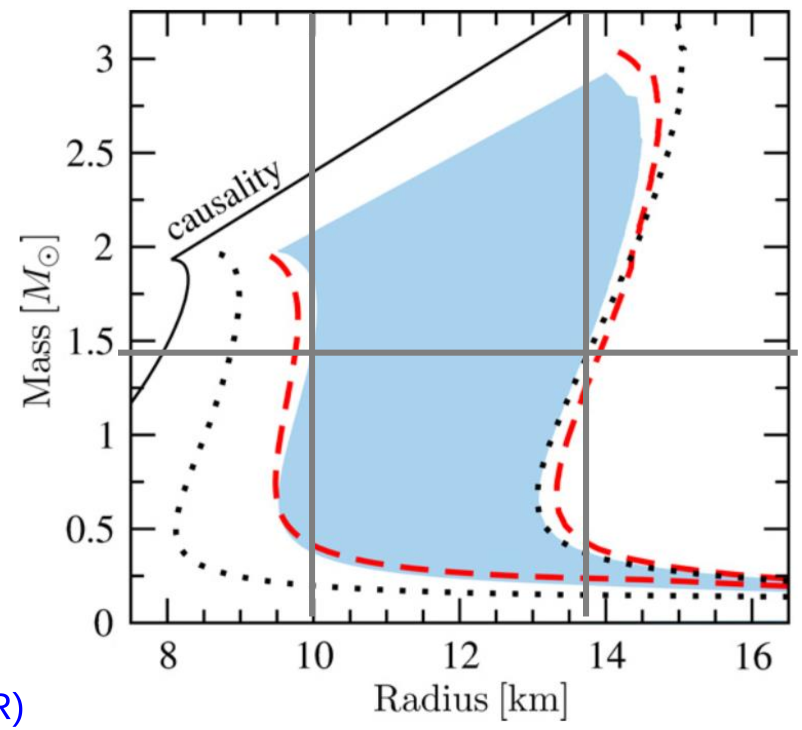


Neutron matter extrapolated to β equilibrium

+ $M_{max} \geq 1.97\ M_{sun}$

+ causality

Mass-Radius Relation



Improvements needed:

- calculate asym. matter directly
- higher orders in the **chiral** and perturbative **expansion**

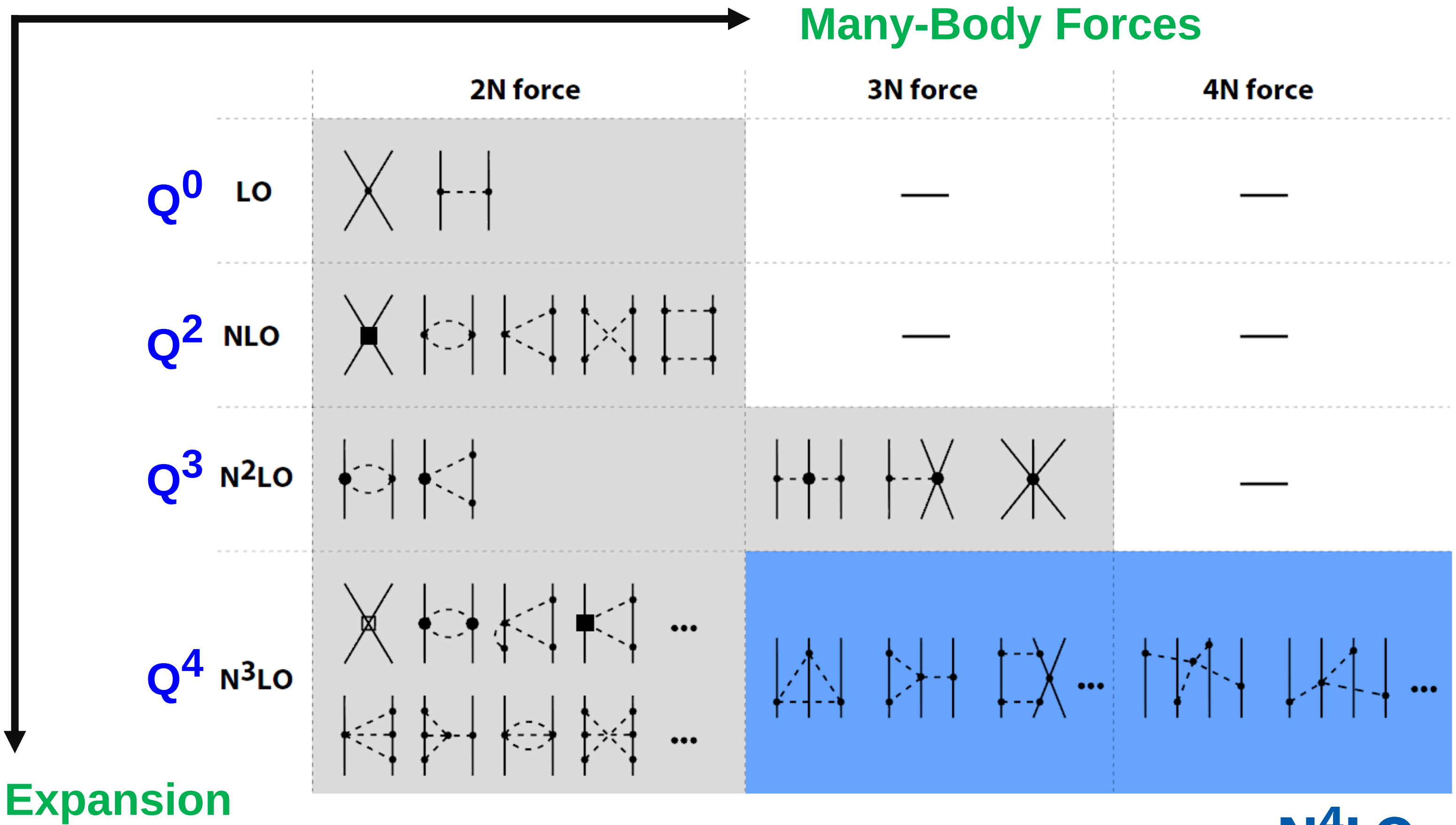
for QMC, see also: Gandolfi *et al.*, Phys. Rev. C **85**, 032801(R)

Applications of chiral nuclear forces up to N^{3}LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Hierarchy of Nuclear Forces in Chiral EFT

see: Epelbaum *et al.*, PRL **115**, 122301



… and ongoing work at N^{4}LO …

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, ...

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

3N forces beyond Hartree-Fock?

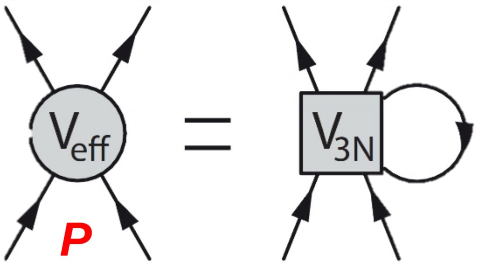
CD, Hebeler, Schwenk, PRC **93**, 054314

Effective NN potentials

by summing *one* particle over the occupied states of the Fermi sea

» dominant 3N contributions

Holt *et al.*, PRC **81**, 024002
Hebeler *et al.*, PRC **82**, 014314



some more applications:
Wellenhofer *et al.*, PRC **92**, 015801
Holt *et al.*, Progr. Part. Nucl. Phys. **73**, 35
Hebeler *et al.*, Ann. Rev. Nucl. Part. Sci. **65**, 457–84
...

so far: only N²LO 3N and ***P*** = 0

Improved Method

- applicable to all nuclear forces
- N³LO 3N forces due to recent PW decomposition

Hebeler *et al.*, PRC **91**, 044001

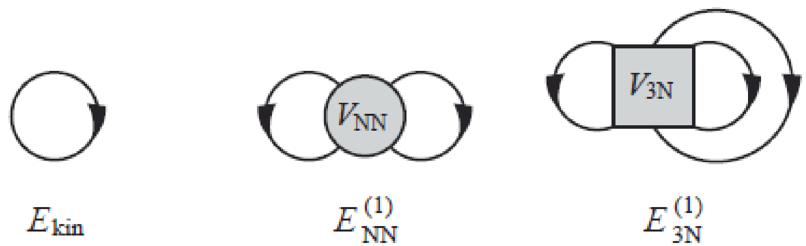
» **towards *consistent* N³LO calculations**

MBPT Diagrams

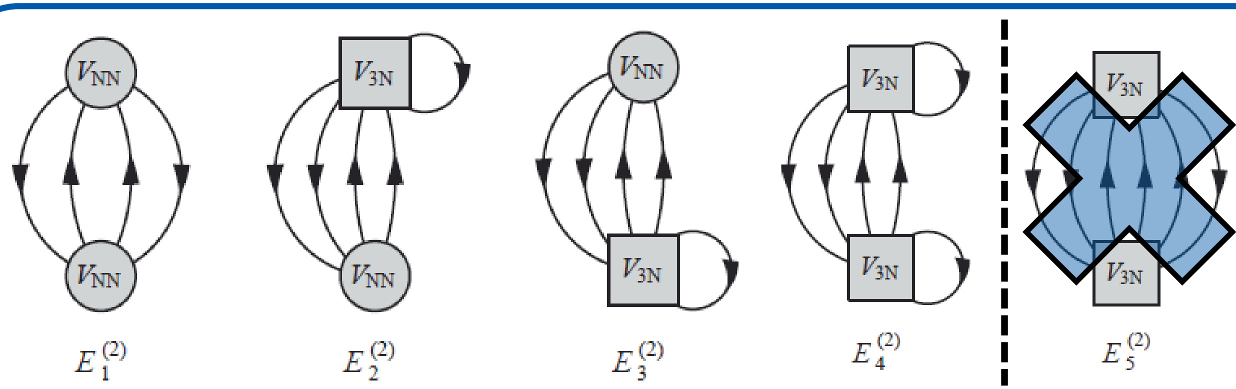
Hebeler *et al.*, Phys. Rev. C **82**, 014314

Normal-Ordered NN level $$\bar{V}_{\text{as}} = V_{\text{NN}} + \xi\, V_{\text{eff}}$$

Initial NN Forces
Normal-Ordered 3N Forces
combinatorial factor



Hartree-Fock



Second Order

+ Higher Orders

TECHNISCHE UNIVERSITÄT DARMSTADT

Outline

0 **Improved Normal-Ordering Method**

1 **Isospin-Asymmetric Nuclear Matter**

2 **Many-Body Convergence?**

3 **BCS Pairing Gaps in Neutron Matter**

$$S_2(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n,\beta)\Big|_{\beta=0}$$

$$L = 3 n_0 \frac{\partial}{\partial n} S_2(n)\Big|_{n=n_0}$$

see also:
Vidaña *et al.*, PRC **80**, 045806
CD *et al.*, PRC **89**, 025806
Drews, Weise, PRC **91**, 035802
Wellenhofer *et al.*, PRC **93**, 055802

CD, Hebeler, Schwenk, PRC **93**, 054314.

ISOSPIN-ASYMMETRIC NUCLEAR MATTER

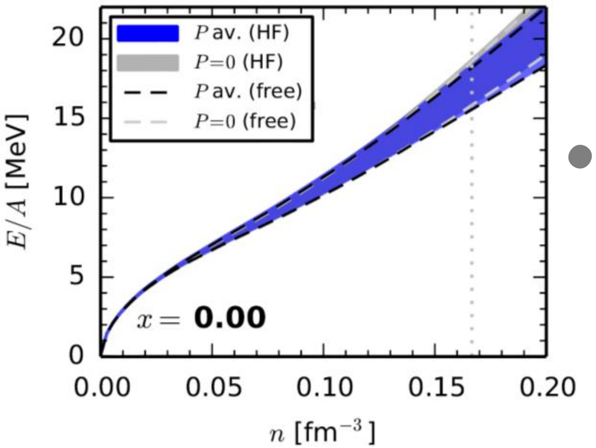
Objectives: equation of state, saturation point, incompressibility, symmetry energy

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

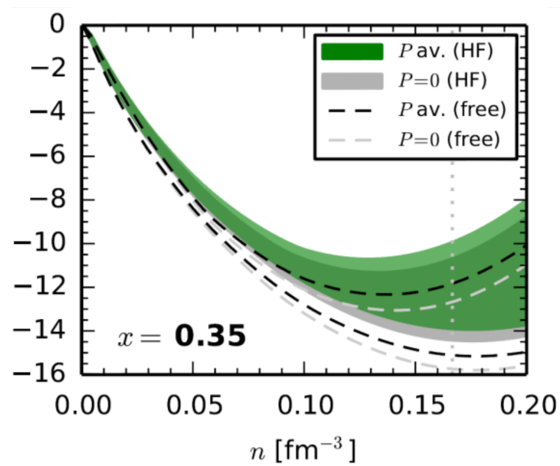
TECHNISCHE UNIVERSITÄT DARMSTADT

Equation of State

CD, Hebeler, Schwenk, PRC **93**, 054314



Neutron Matter



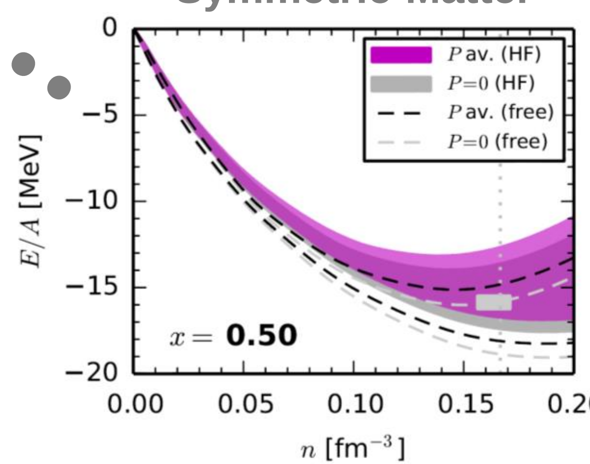
11 proton fractions

***x* = 0.0, 0.05, ..., 0.5**

up to second order

$$x = \frac{n_p}{n_n + n_p}$$

Symmetric Matter



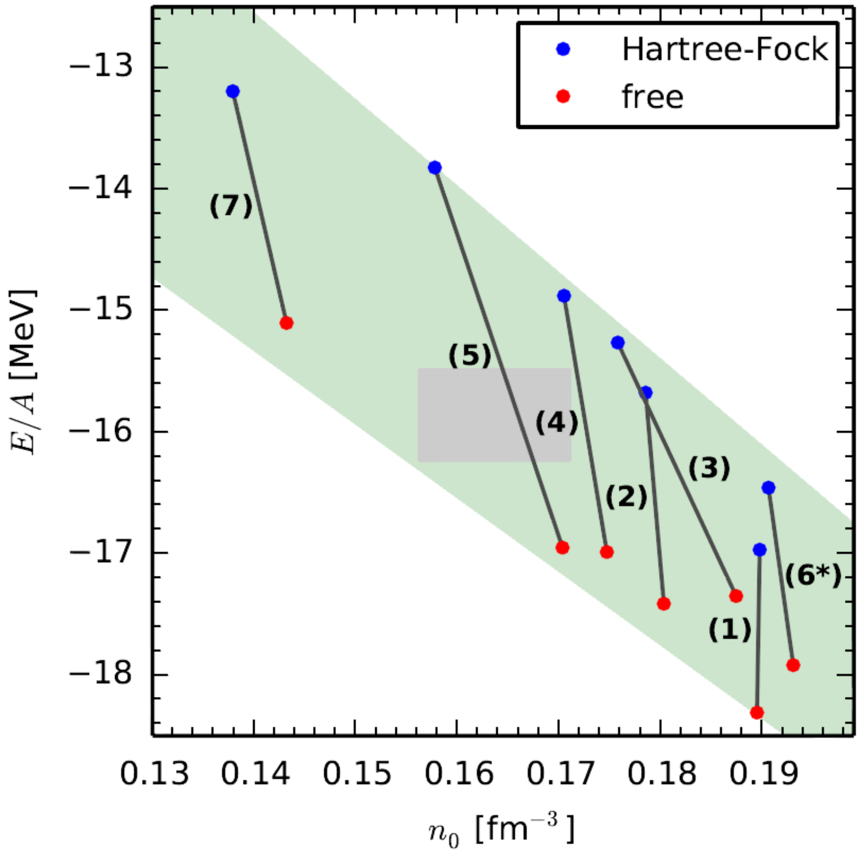
Uncertainty bands: Hebeler *et al.*, PRC **83**, 031301(R)

- 7 Hamiltonians: evolved N^3LO NN + bare N^2LO 3N
- different combinations of λ/Λ_{3N}
- c_D, c_E fit *only* to few-body data
- free and Hartree-Fock spectrum

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

Saturation Properties

CD, Hebeler, Schwenk, PRC **93**, 054314



Coester-like correlation

- covers the empirical range due to 3N contributions

Coester *et al.*, PRC **1**, 769

empirical saturation point:

max. range of 14 EDF's

Dutra *et al.*, PRC **85**, 035201
Kortelainen *et al.*, PRC **89**, 054314

$$n_0 = (0.138 - 0.193)\ \mathrm{fm}^{-3}$$

$$K = (182 - 254)\ \mathrm{MeV}$$

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

Symmetry Energy and Slope Parameter

see also: Hagen *et al.*, Nat. Phys. **12**, 186

standard expansion: $\beta = 1 - 2x$

$$\frac{E}{A}(n,\beta) = \frac{E_{\mathrm{SNM}}(n)}{A} + S_2(n)\beta^2 + \ldots$$

$$S_2(n) = S_v + \frac{L}{3}\left(\frac{n-n_0}{n_0}\right) + \cdots$$

tight constraints

$$S_v = (30.9 \pm 1.4)\ \mathrm{MeV}$$

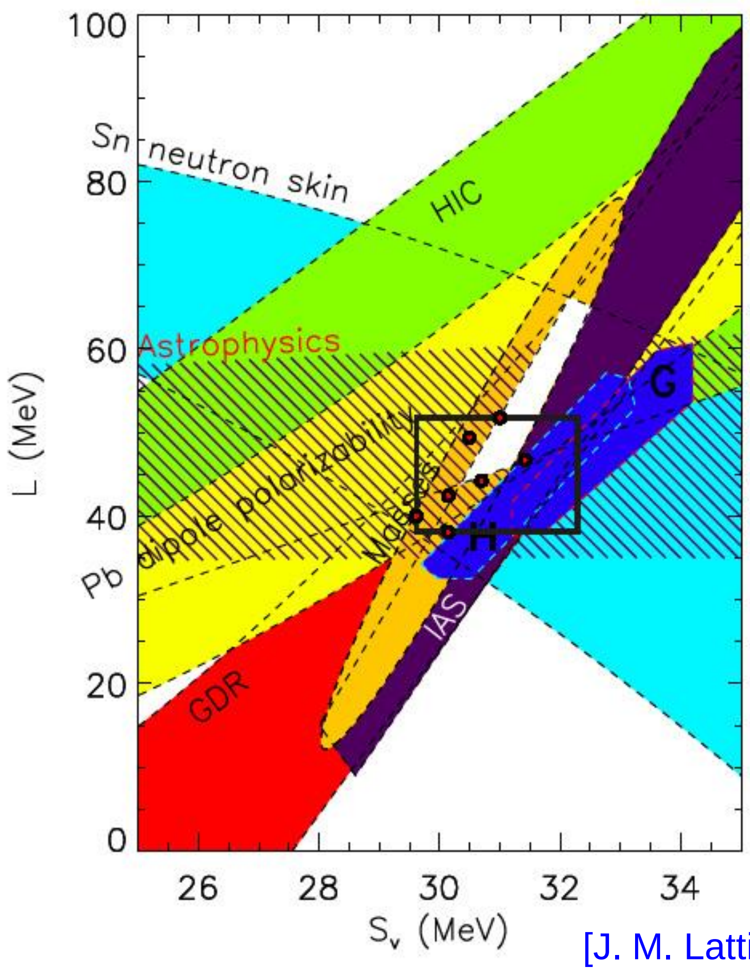
$$L = (45.0 \pm 7.1)\ \mathrm{MeV}$$

in **agreement** with emp. extractions

Lattimer, Lim, Astrophys. J. **771**, 51

quadratic expansion is **reliable**;
but nonanalytical quartic term: $\beta^4 \ln|\beta|$

Kaiser, PRC **91**, 065201
Wellenhofer *et al.*, PRC **93**, 055802



[J. M. Lattimer]

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

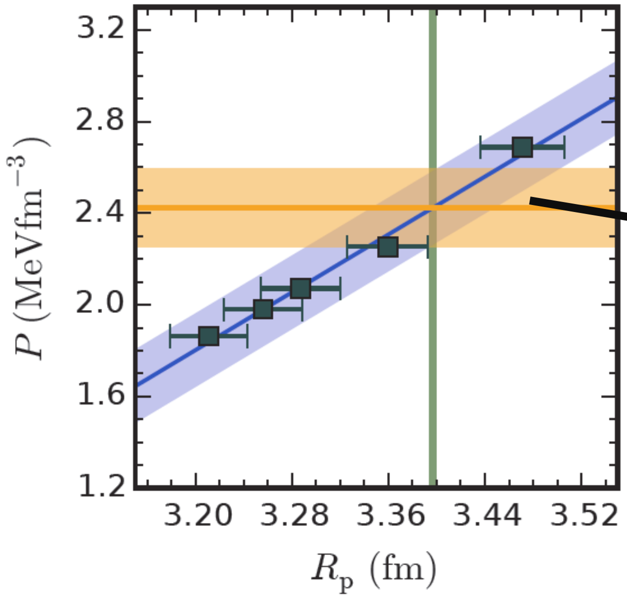
TECHNISCHE UNIVERSITÄT DARMSTADT

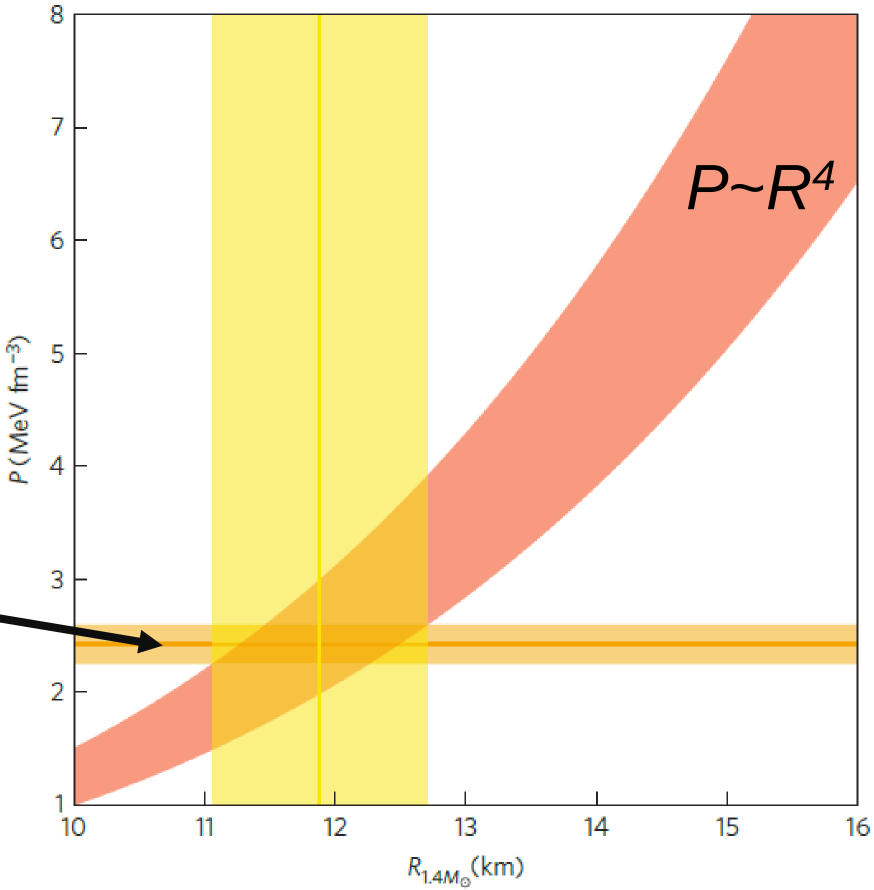
Radius Estimate for a 1.4 M_{sun} Neutron Star

Hagen *et al.*, Nat. Phys. **12**, 186

Universal (empirical) relation by Lattimer, Prakash

Pressure constrained by CC calculations of ^{48}Ca radii and measurement



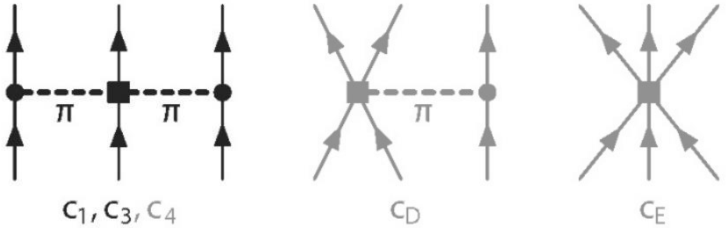


Lattimer, Prakash, APJ, **550**, 426
Lattimer, Lim, Astrophys. J. **771**, 51

see also:
Dickhoff, Barbieri, Prog. Part. Nucl. Phys. **52**, 377
Rios *et al.*, PRC **79**, 025802
Krüger *et al.*, PRC **88**, 025802
Tews *et al.*, PRC **93**, 024305

TECHNISCHE UNIVERSITÄT DARMSTADT

Neutron Matter:



CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307.

MANY-BODY CONVERGENCE?

Objectives: test many-body convergence
study impact of N^3LO 3N forces

Weinberg eigenvalue analysis: Hoppe, CD, Furnstahl, Hebeler, Schwenk, in prep.

Applications of chiral nuclear forces up to N^{3}LO to nuclear matter and neutron stars

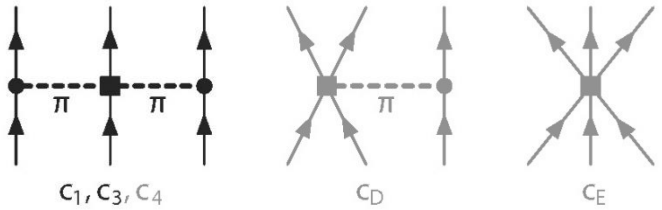
Testing Many-Body Convergence

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307

- consistent N^{3}LO NN/3N forces
- **finite proton fractions** need reliable fits of c_D, c_E at N^{3}LO

Golak *et al.*, Eur. Phys. J. A **50** 177

Neutron Matter



Uncertainty bands

- use always c_i's recommended for N^{3}LO calculations
- plus many-body uncertainty

Krebs *et al.*, PRC **85**, 054006

MBPT	SCGF Method
Improved Normal-Ordering Method	
up to third order	**nonperturbative**
free vs. HF spectrum	full spectral function
T=0 MeV	Extrapolated to T=0 MeV

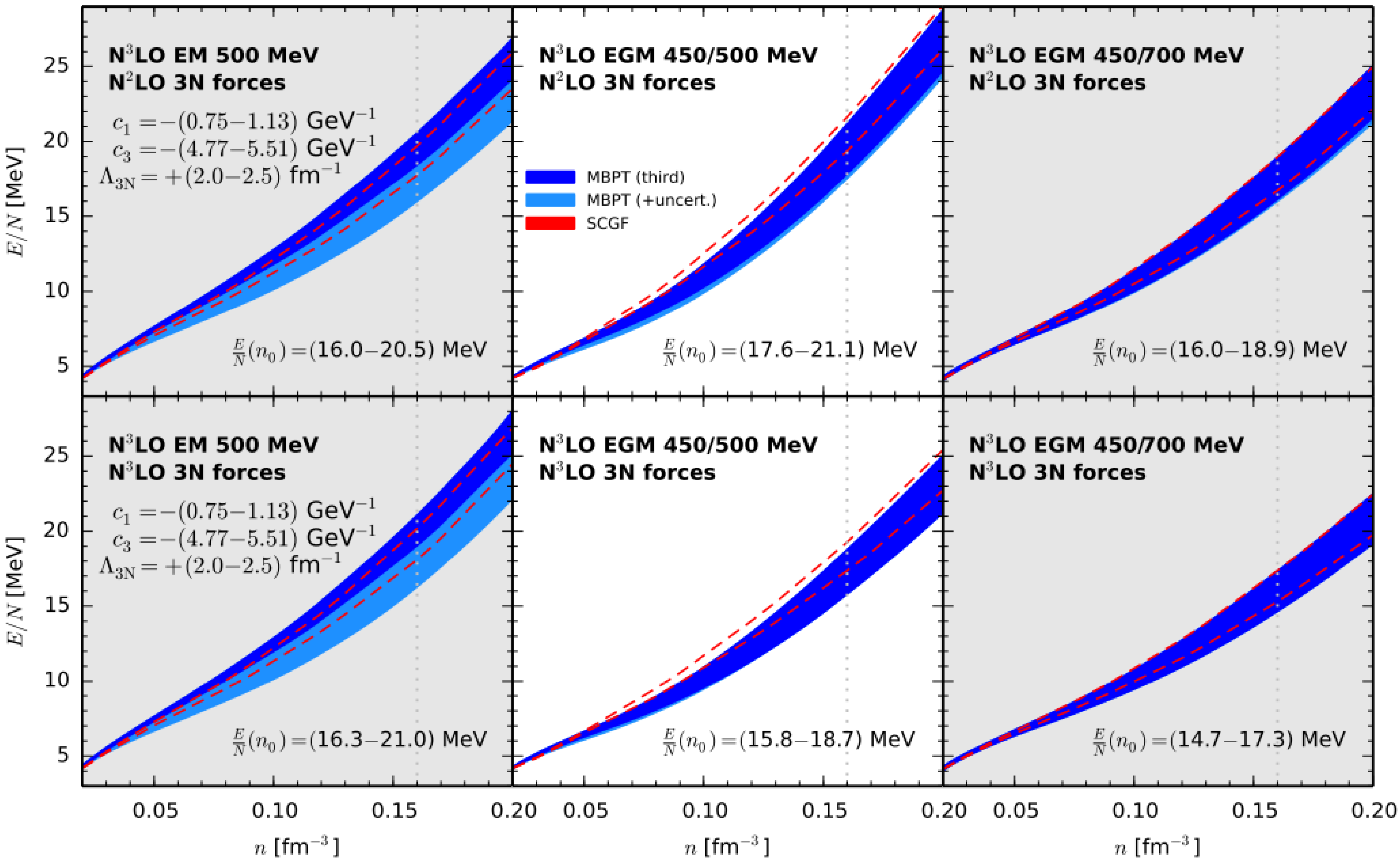
see also: Carbone *et al.*, PRC **90** 054322

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

MBPT vs. SCGF Method

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



N^2LO 3N

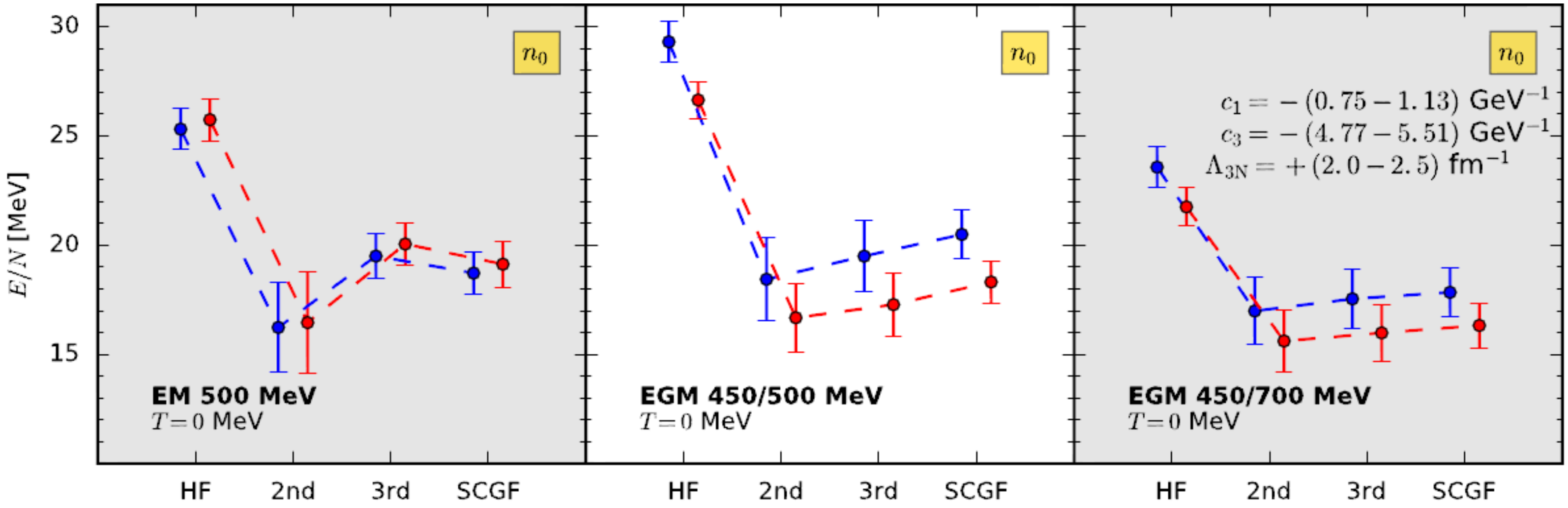
N^3LO 3N

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Testing Many-Body Convergence

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



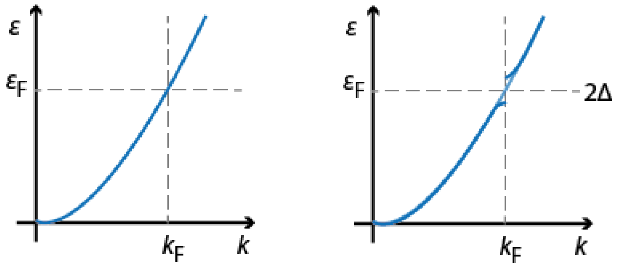
Order-by-order analysis: (at saturation density)

- attractive second vs. repulsive third order
- MBPT **well converged** for EGM potentials (small third order)
- EM 500 MeV is less perturbative (larger third order)
- small energy shift due to N^3LO 3N w.r.t. N^2LO 3N contributions

see also:
Srinivas, Ramanan, PRC **94**, 064303
Ding *et al.*, PRC **94**, 025802
Maurizio *et al.*, PRC **90**, 044003
Page *et al.*, “Novel Superfluids“, Oxford University Press

TECHNISCHE UNIVERSITÄT DARMSTADT

Normal Fermi liquid **Superfluid Fermions**



CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302.

BCS PAIRING GAPS IN NEUTRON MATTER

Objectives: study subleading 3N contributions
recent (semi-)local NN potentials, new uncertainties

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Chiral NN Potentials and Regularization

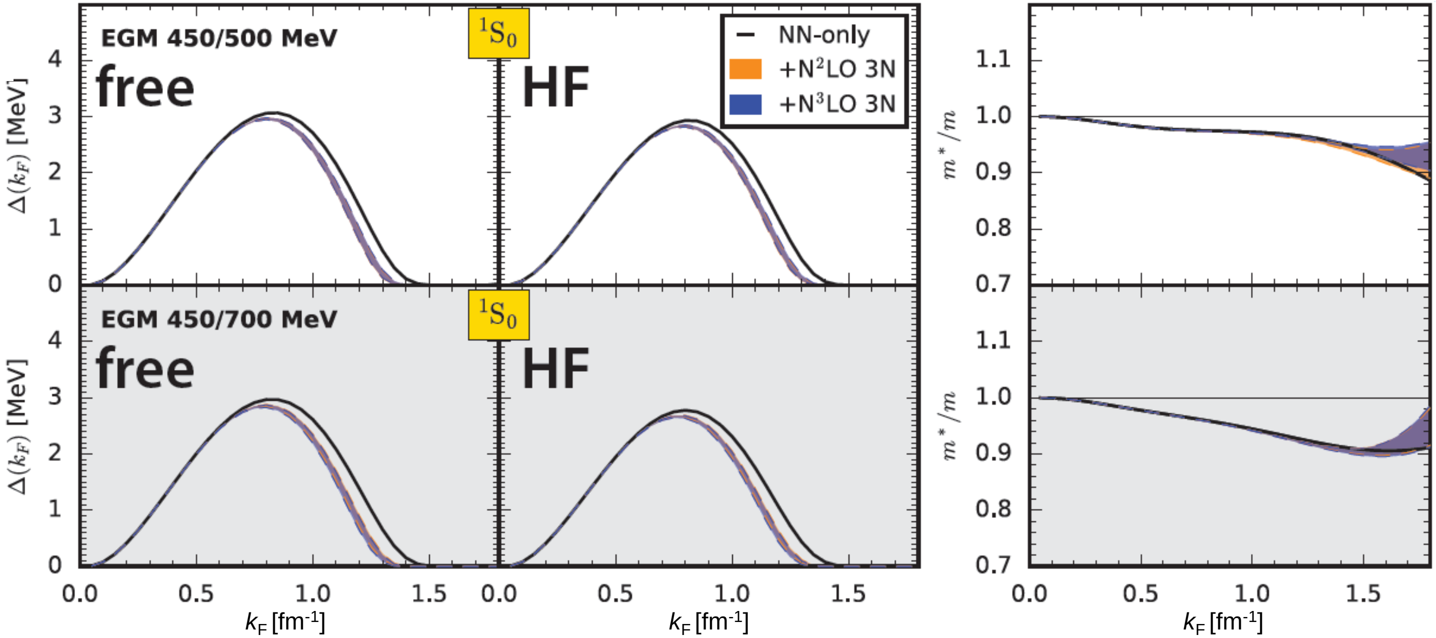
	Short Range	Long Range	Potentials
Nonlocal	Nonlocal		e.g., EM, EGM; Carlsson *et al.*, *PRX **6**, 011019 (2016)*
Local	Local		Gezerlis *et al.*, *PRL **111**, 122301 (2013)*
Semilocal	Nonlocal	Local	Epelbaum *et al.*, *EPJ A **51**, 53 (2015), PRL **115**, 122301 (2015)*

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Pairing Gaps: 3N forces in $^1S_0$

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302



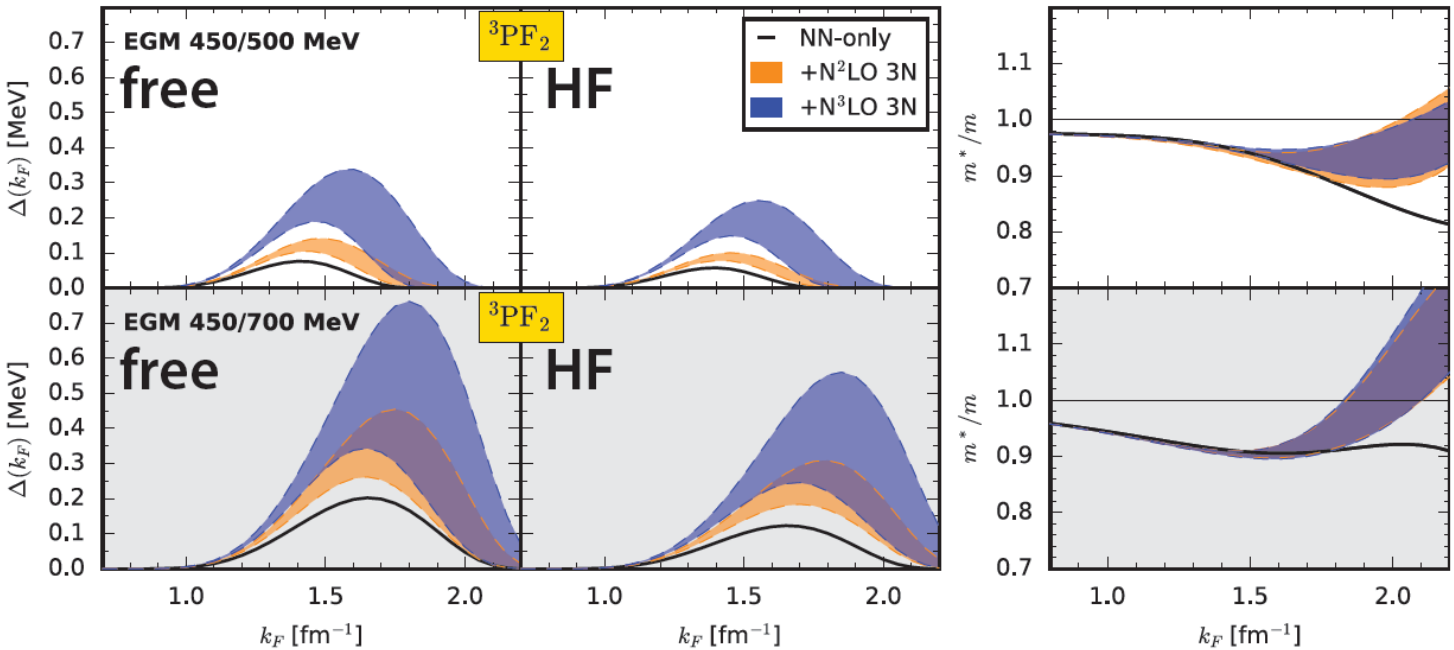
- uncertainties: 3N parameter variation (recommended values)
- **pairing gap at low densities**
  - universal gaps: strongly constrained by phase shifts
  - **small 3N contributions**: only small suppression for $k_F > 0.8$ fm^{-1}
  - almost independent of the energy spectrum

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Pairing Gaps: 3N forces in $^3P_2$–$^3F_2$

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302

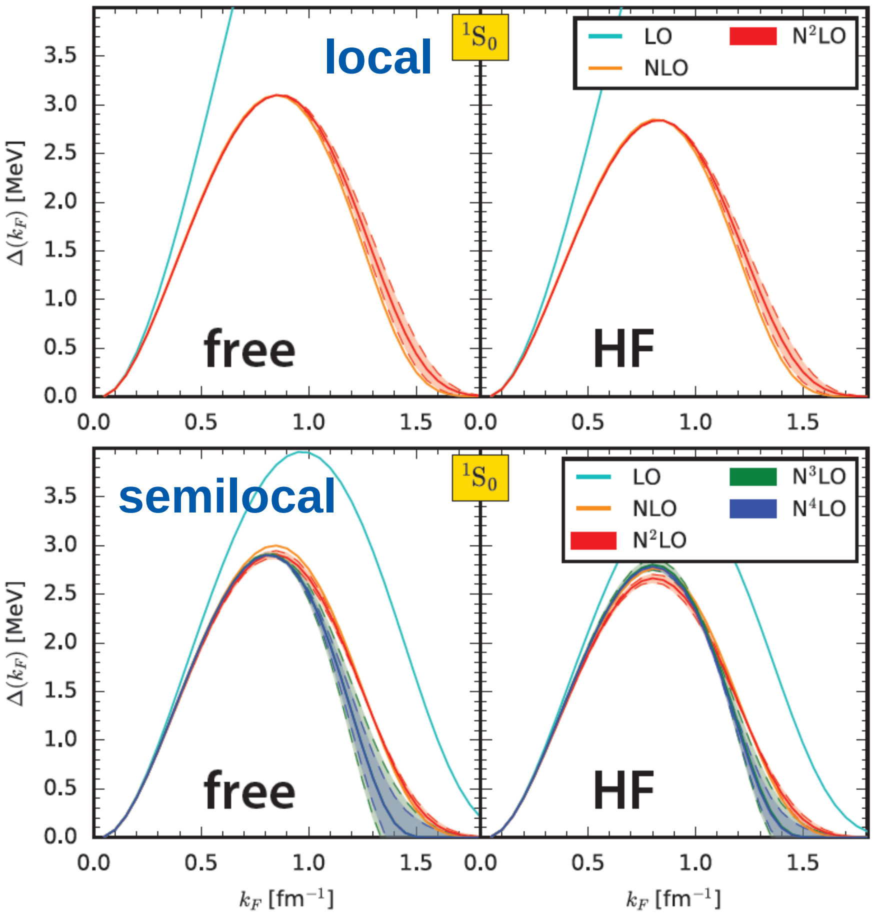


- uncertainties: 3N parameter variation (recommended values)
- **pairing gap at high densities**
  - **3N forces add attraction**: larger max. gap and closure at higher densities
  - effective masses are enhanced due to 3N forces
  - chiral EFT still efficient at $k_F > 2$ fm^{-1}**?**

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

(Semi-)Local NN: $^1S_0$ channel

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302



local and semilocal NN forces:

- up to N^2LO and N^4LO
- R_0 = 0.9, 1.0, 1.1 and, 1.2 fm

new uncertainties (Epelbaum *et al.*)

order-by-order analysis in the chiral expansion (LO neglected)

findings: $Q(k_\mathrm{F}) = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right)$

- at NLO and beyond **gaps agree** up to $k_\mathrm{F} \sim (0.6\text{–}0.8)\ \mathrm{fm}^{-1}$
- sensitivity to spectrum is again small

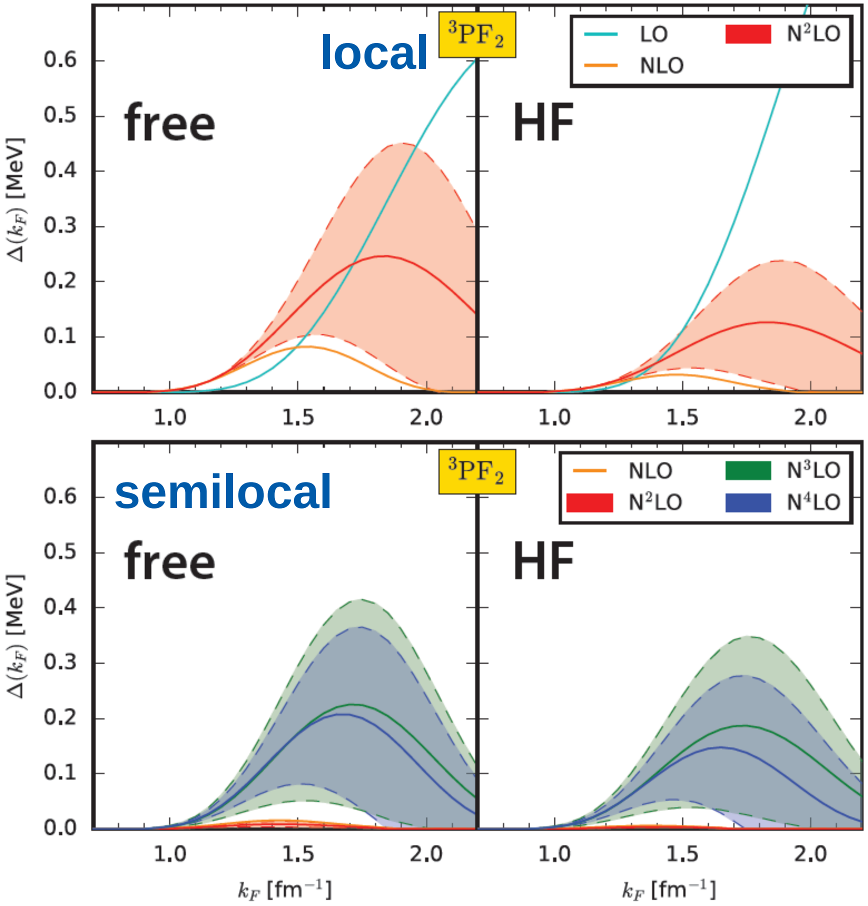
Gezerlis *et al.*, PRC **90**, 054323
Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

(Semi-)Local NN: $^3P_2$–$^3F_2$ channel

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302



local and semilocal NN forces:

- up to N^2LO and N^4LO
- R_0 = 0.9, 1.0, 1.1 and, 1.2 fm

new uncertainties (Epelbaum *et al.*)

order-by-order analysis in the chiral expansion (LO neglected)

findings:

- large uncertainties: breakdown of the chiral expansion **?**

Gezerlis *et al.*, PRC **90**, 054323

Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Summary | Outlook

Improved Normal-Ordering Method

- applicable to all 3N forces (incl. N^3LO)
- **asymmetric matter**: results for EOS, symmetry energy, …

More Applications

- studied **many-body convergence** in neutron matter: N^3LO 3N forces beyond Hartree-Fock and in SCGF method
- **BCS pairing gaps** in $^1S_0$ and $^3P_2$–$^3F_2$:
  - N^3LO 3N contributions to previous NN potentials
  - recent (semi-)local NN potentials, new uncertainties

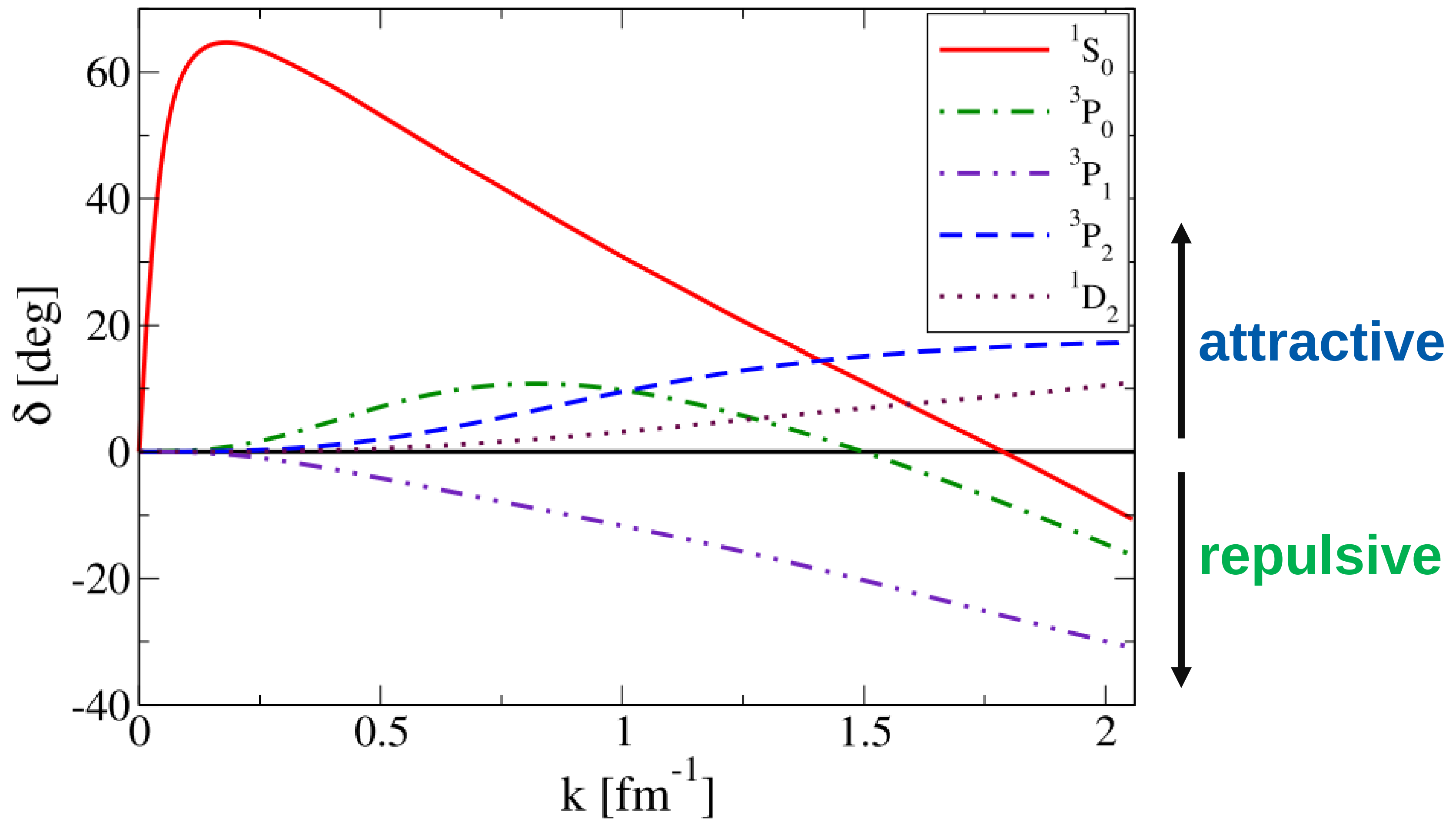
Extensions – a selection

- finite temperatures, consistently-evolved forces, ...
- **constrain next-gen. potentials: saturation**, … Carlsson *et al.*, PRX **6**, 011019

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Attractive Interactions: Phase Shifts



Pairing in neutron matter:
New uncertainty estimates and 3N forces

The Gap Equation

$$\Delta^{J}_{lS}(k) = -\int_0^\infty \frac{dk'\, k'^2}{\pi} \sum_{l'} \frac{i^{l'-l} V^{J}_{ll'S}(k,k')\, \Delta^{J}_{l'S}(k')}{\sqrt{(\varepsilon_{k'}-\mu)^2 + \sum_{\tilde{l},\tilde{S},\tilde{J}} |\Delta^{\tilde{J}}_{\tilde{l}\tilde{S}}(k')|^2}}$$

Pairing in neutron matter: New uncertainty estimates and 3N forces

New Uncertainties

Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

$$\Delta X^{\mathrm{N^3LO}}(p) = \max\Big(Q^5 \times \big|X^{\mathrm{LO}}(p)\big|,$$
$$Q^3 \times \big|X^{\mathrm{LO}}(p) - X^{\mathrm{NLO}}(p)\big|,$$
$$Q^2 \times \big|X^{\mathrm{NLO}}(p) - X^{\mathrm{N^2LO}}(p)\big|,$$
$$Q \times \big|X^{\mathrm{N^2LO}}(p) - X^{\mathrm{N^3LO}}(p)\big|\Big),$$

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

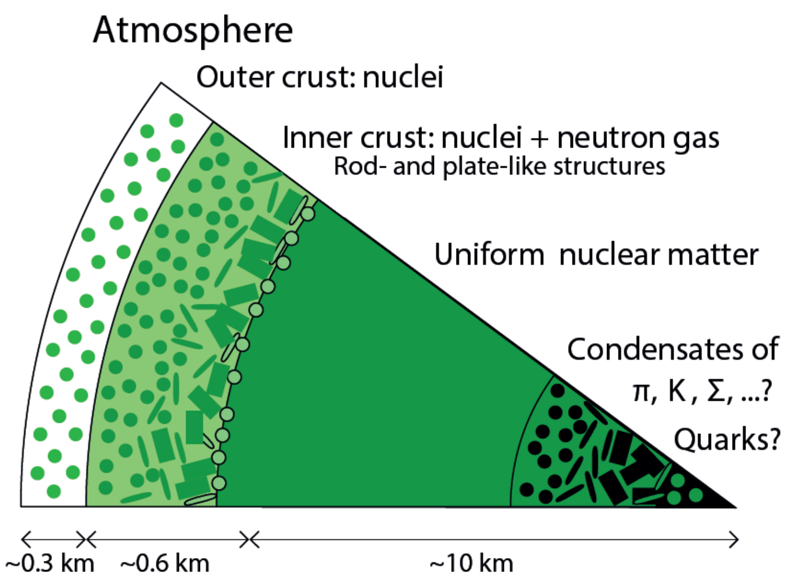
References for Asymmetric Matter Calculations

Calculations (variational, BHF, SCGF, IM-ChPT...)
Fiorilla, Kaiser, Holt, Weise, (2002-12)
Frick *et al.*, PRC **71**, 014313 (2005)
Vidaña *et al.*, PRC **80**, 045806 (2009)
Oller *et al.*, J. Phys. G: NPP **37**, 015106 (2009)
Drischler *et al.* PRC **89**, 025806 (2014)
Drews, Weise PRC **91**, 035802 (2015)
Wellenhofer *et al.,* PRC **93**, 055802
Kaiser, PRC **91**, 065201 (2015)
…

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Motivation: Neutron Stars



Lattimer, Prakash, Science **304**, 536 (2004)

neutron stars are of extremes:

- R ~ (10 − 14) km, M ~ 2 M_{sun}
- most densest objects we observe

outer core: $n \sim n_0$

- homogeneous, infinite nuclear matter

nuclear matter: well-suited system to apply/check

- nuclear forces
- many-body approaches

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Chiral Effective Field Theory

Nuclear Matter interacts via the Strong Interaction

(not considering Coulomb)

- fundamental theory is known
- QCD is non-perturbative at low energies of interest
- **modern approach**: chiral EFT
  - relevant degrees of freedom instead of quarks/gluons
  - use e.g., **nucleons** and **pions**
  - pion exchanges and short-range contact interactions
  - expand most general Lagrangian in powers of $Q = max(\, p, m_\pi) \,/\, \Lambda_\mathrm{b} \sim 1/3$

Weinberg, Phys. Lett. B **251**, 288 (1990)
Weinberg, NP B **363**, 3 (1991)
Weinberg, Phys. Lett. B **295**, 114 (1992)

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Neutron Stars in β Equilibrium: $0 < x \ll 0.5$

Such calculations are more involved: less symmetries

$$x = \frac{n_p}{n_p + n_n} \qquad \text{or,} \qquad \beta = \frac{n_n - n_p}{n_n + n_p} \qquad \text{with} \quad \beta = 1 - 2x$$

Obtaining the equation of state:

- parametrizations (fits to PNM plus empirical properties)
- empirically constrained coefficients

$$\frac{E}{A}(\beta, n) \overset{\text{Taylor}}{=} \sum_{i,j} C_{ij}\, \beta^i \left(\frac{n - n_0}{n_0} \right)^j$$

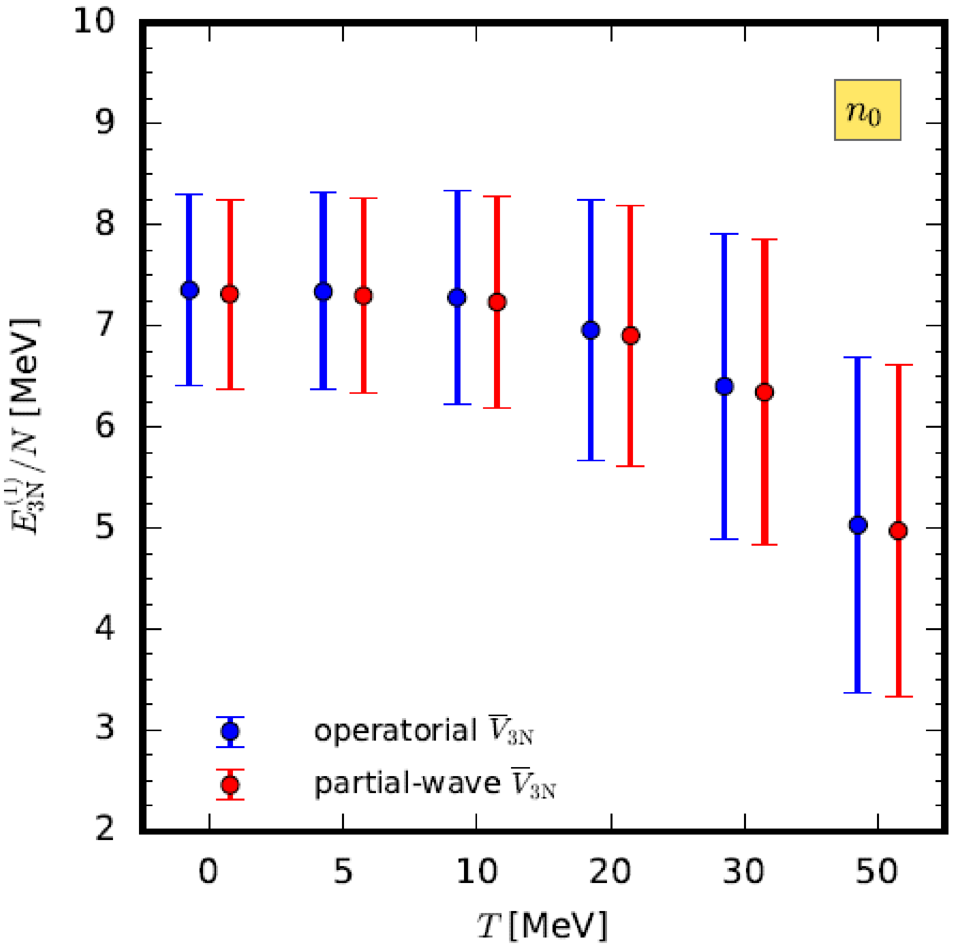
e.g., $C_{00} \sim -16$ MeV, $C_{20} \sim 31$ MeV, …

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Normal-Ordering at Finite T

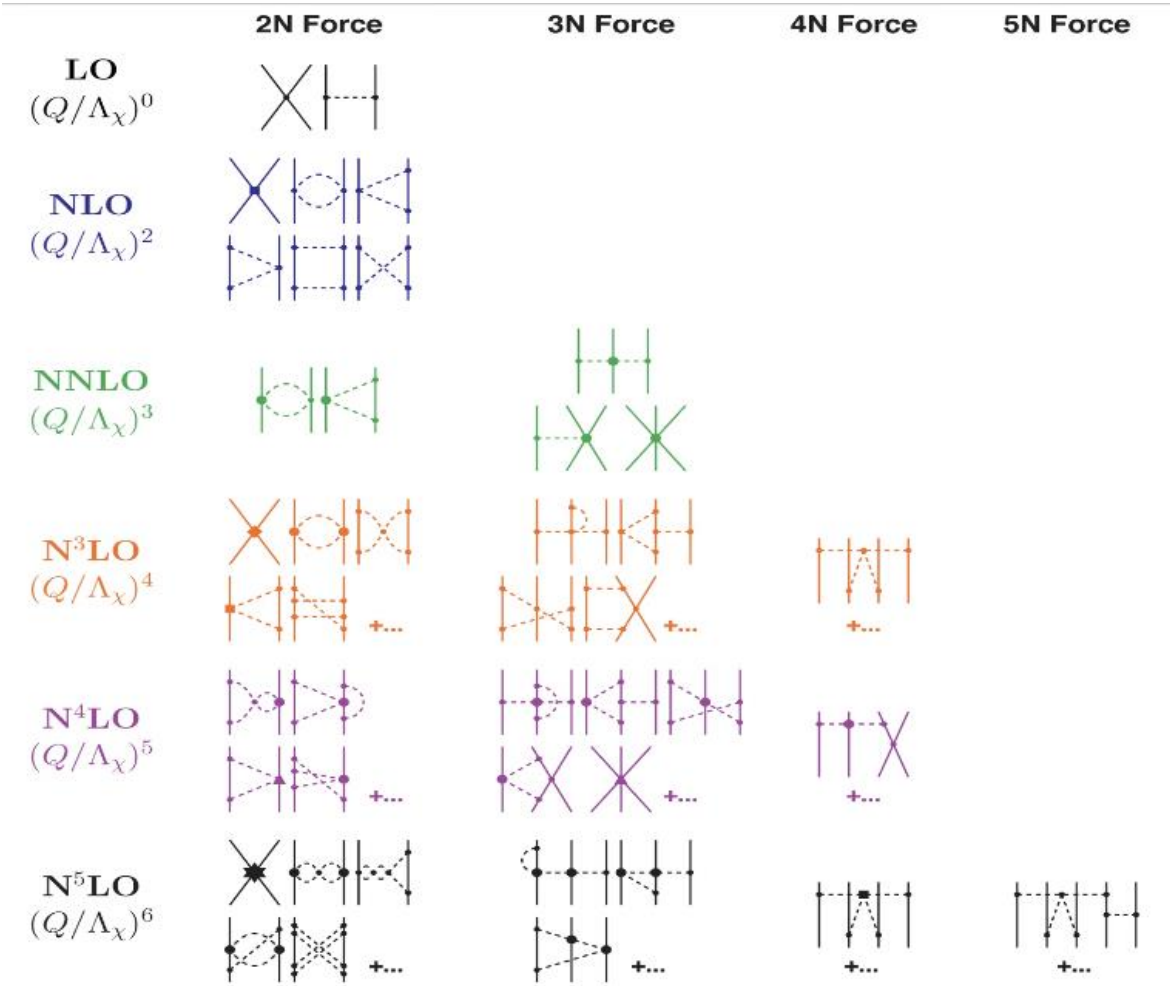
CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



Improved Normal-ordering works also at finite T

Applications of chiral nuclear forces up to N³LO to nuclear matter and neutron stars

Hierachy of Chiral EFT



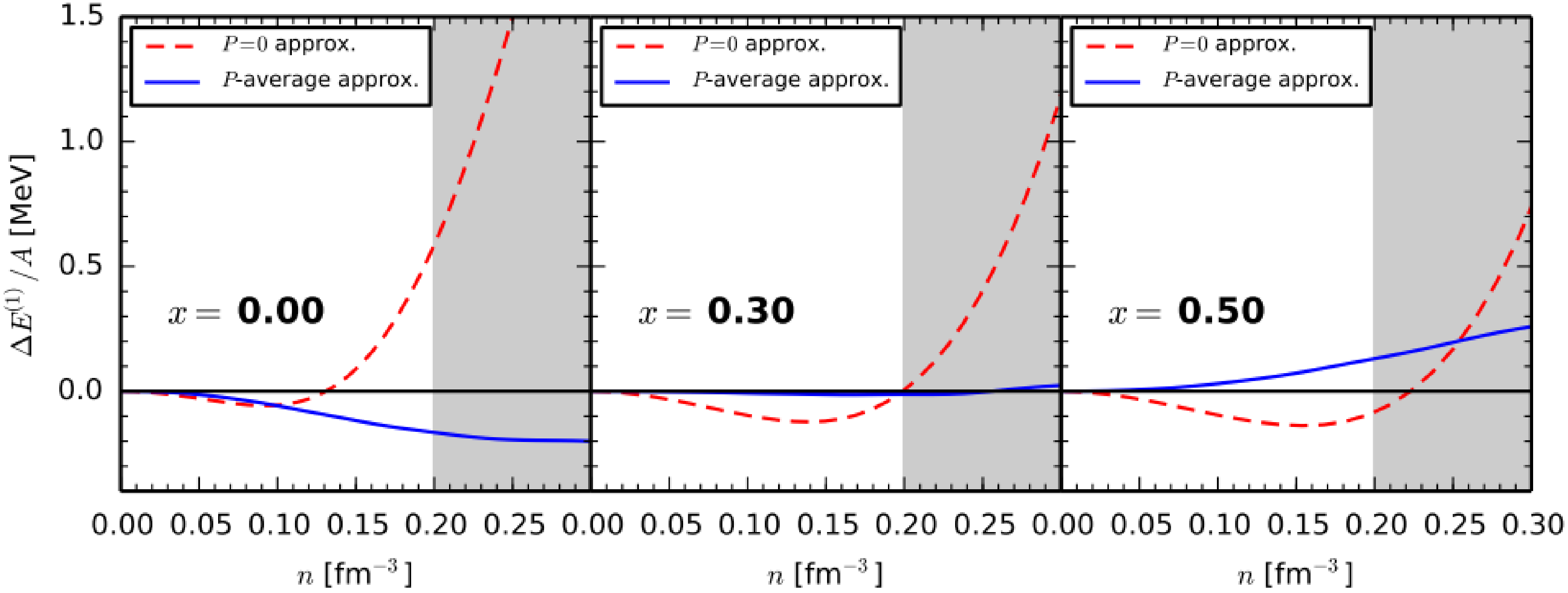
Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

TECHNISCHE UNIVERSITÄT DARMSTADT

Eff. NN potential vs. full 3N force

CD, Hebeler, Schwenk, PRC **93**, 054314

Compare HF energies:

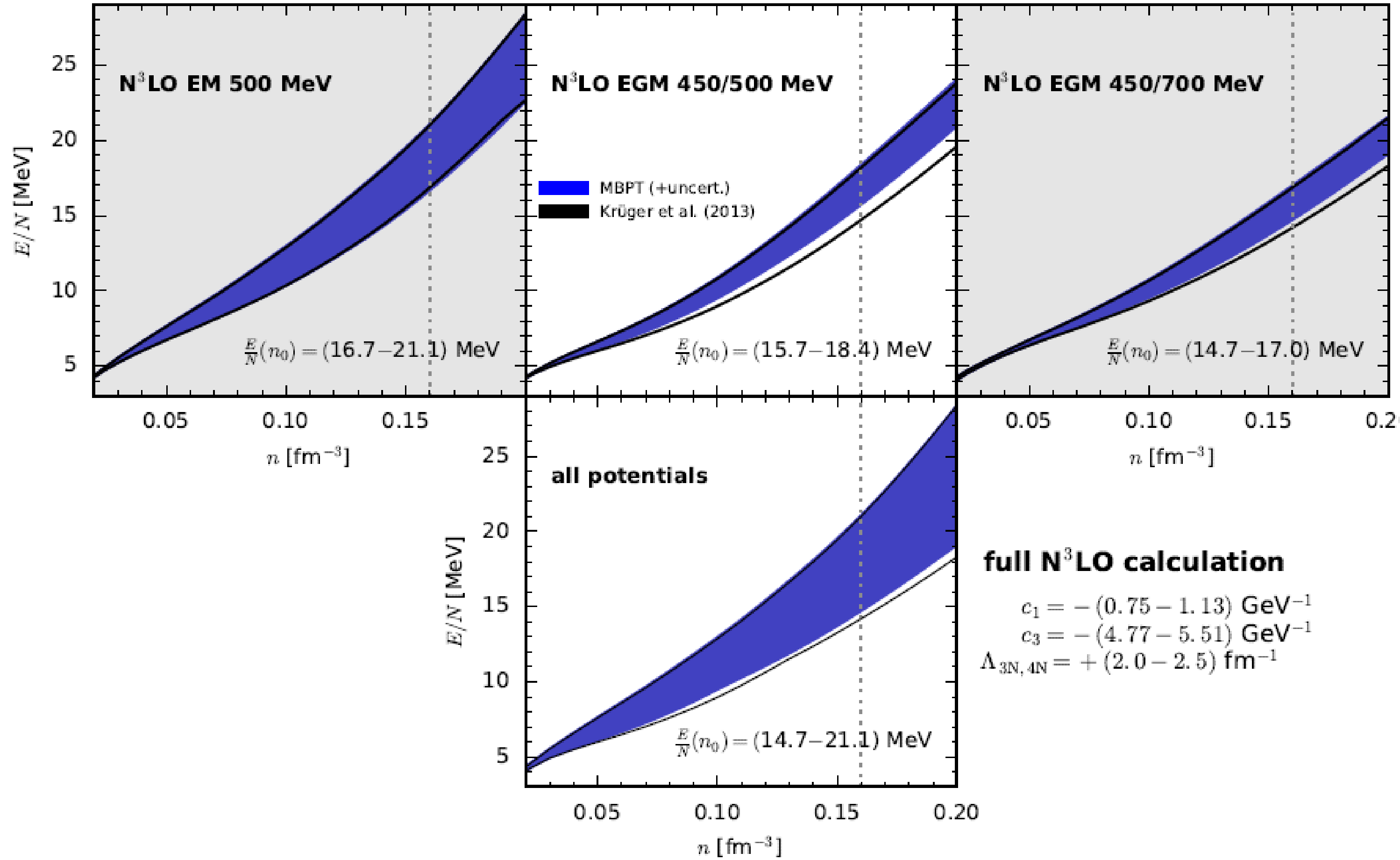


significant improvement: *P*-av. method, in particular for $n >> n_0$

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

Full N^3LO calculations

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



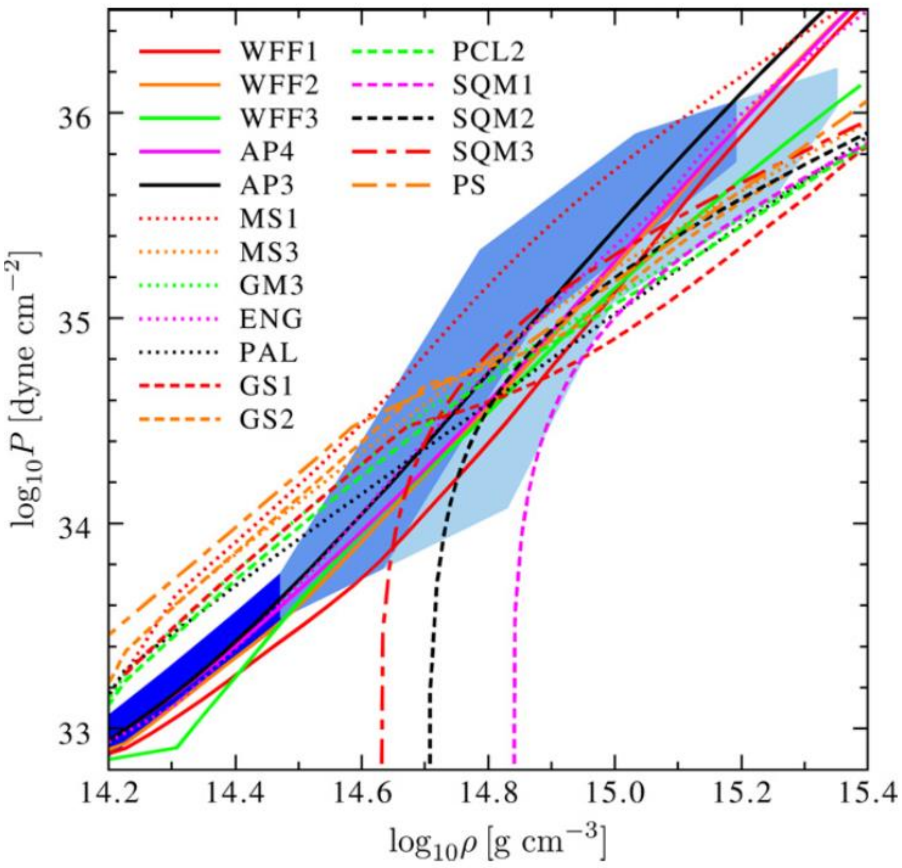
full N^3LO calculation

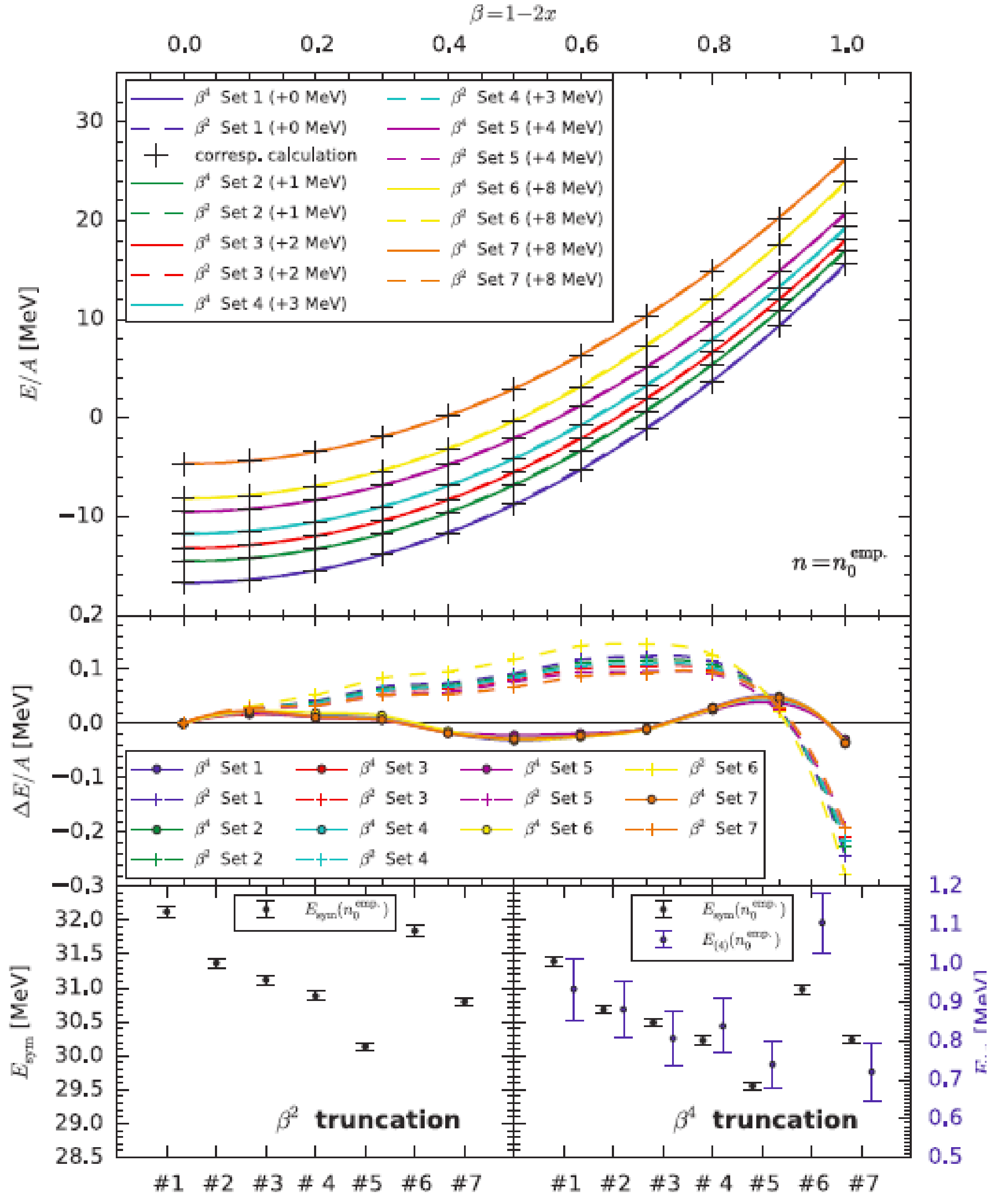
$c_1 = -(0.75 - 1.13)$ GeV^{-1}
$c_3 = -(4.77 - 5.51)$ GeV^{-1}
$\Lambda_{3N,4N} = +(2.0 - 2.5)$ fm^{-1}

Applications of chiral nuclear forces up to N^3LO to nuclear matter and neutron stars

Constraints on the EOS

Hebeler *et al.*, Astrophys. J., **773**, 11





$\beta = 1-2x$
0.0
0.2
0.4
0.6
0.8
1.0
β^4 Set 1 (+0 MeV)
β^2 Set 1 (+0 MeV)
corresp. calculation
β^4 Set 2 (+1 MeV)
β^2 Set 2 (+1 MeV)
β^4 Set 3 (+2 MeV)
β^2 Set 3 (+2 MeV)
β^4 Set 4 (+3 MeV)
β^2 Set 4 (+3 MeV)
β^4 Set 5 (+4 MeV)
β^2 Set 5 (+4 MeV)
β^4 Set 6 (+8 MeV)
β^2 Set 6 (+8 MeV)
β^4 Set 7 (+8 MeV)
β^2 Set 7 (+8 MeV)
E/A [MeV]
30
20
10
0
−10
$n = n_0^{\text{emp.}}$
0.2
0.1
0.0
−0.1
−0.2
−0.3
$\Delta E/A$ [MeV]
β^4 Set 1
β^2 Set 1
β^4 Set 2
β^2 Set 2
β^4 Set 3
β^2 Set 3
β^4 Set 4
β^2 Set 4
β^4 Set 5
β^2 Set 5
β^4 Set 6
β^2 Set 6
β^4 Set 7
β^2 Set 7
E_{sym} [MeV]
32.0
31.5
31.0
30.5
30.0
29.5
29.0
28.5
$E_{\text{sym}}(n_0^{\text{emp.}})$
β^2 truncation
$E_{\text{sym}}(n_0^{\text{emp.}})$
$E_{(4)}(n_0^{\text{emp.}})$
β^4 truncation
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
#1 #2 #3 # 4 #5 #6 #7
#1 #2 #3 # 4 #5 #6 #7